A Fault Diagnosis Method for Industrial Gas Turbines Using Bayesian Data Analysis

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This paper presents an offline fault diagnosis method for industrial gas turbines in a steady-state. Fault diagnosis plays an important role in the efforts for gas turbine owners to shift from preventive maintenance to predictive maintenance, and consequently to reduce the maintenance cost. Ever since its birth, numerous techniques have been researched in this field, yet none of them is completely better than the others and perfectly solves the problem. Fault diagnosis is a challenging problem because there are numerous fault situations that can possibly happen to a gas turbine, and multiple faults may occur in multiple components of the gas turbine simultaneously. An algorithm tailored to one fault situation may not perform well in other fault situations. A general algorithm that performs well in overall fault situations tends to compromise its accuracy in the individual fault situation. In addition to the issue of generality versus accuracy, another challenging aspect of fault diagnosis is that, data used in diagnosis contain errors. The data is comprised of measurements obtained from gas turbines. Measurements contain random errors and often systematic errors like sensor biases as well. In this paper, to maintain the generality and the accuracy together, multiple Bayesian models tailored to various fault situations are implemented in one hierarchical model. The fault situations include single faults occurring in a component, and multiple faults occurring in more than one component. In addition to faults occurring in the components of a gas turbine, sensor biases are explicitly included in the multiple models so that the magnitude of a bias, if any, can be estimated as well. Results from these multiple Bayesian models are averaged according to how much each model is supported by data. Gibbs sampling is used for the calculation of the Bayesian models. The presented method is applied to fault diagnosis of a gas turbine that is equipped with a faulty compressor and a biased fuel flow sensor. The presented method successfully diagnoses the magnitudes of the compressor fault and the fuel flow sensor bias with limited amount of data. It is also shown that averaging multiple models gives rise to more accurate and less uncertain results than using a single general model. By averaging multiple models, based on various fault situations, fault diagnosis can be general yet accurate. [DOI: 10.1115/1.3204508]

1 Introduction

As the power generation market becomes competitive, power plant owners strive to make more profit with lesser cost of ownership. Maintenance cost accounts for a large part of the cost of ownership. The current maintenance strategy for most machines is *preventive* in a sense that maintenance actions are performed along schedules suggested by manufacturers. These schedules are made by the manufacturers, based on historical data, empirical knowledge, and tests performed along design processes. The schedules have little to do with the actual condition of the machine subject to the scheduled maintenance actions. To reduce the maintenance cost, it is desirable for the power plant owners to perform maintenance actions when they are actually needed. This desire has led to a new maintenance strategy called *predictive* maintenance, with which maintenance experts assess the condition of machines at the current time, predict the failure time in the future, and decide the best maintenance action. The first two steps are called *fault diagnosis* and *prognosis*, respectively. For the predictive maintenance to be successful, it is important for a diagnosis to be accurate because not only a wrong diagnosis results in unnecessary maintenance and consequently high maintenance cost but also diagnosis results are used in prognosis and other tasks downstream.

Fault diagnosis and prognosis are not new concepts in the gas turbine industry for power plants. A gas turbine is a crucial component of conventional combined cycle power plants so it has been of great interest for power plant operators to estimate the condition of a gas turbine from tests or operation data. The condition of a gas turbine is quantitatively represented by, commonly called, *health parameters*, which scales gas turbine performance relative to a baseline, e.g., the performance of a brand new gas turbine. Health parameters are immeasurable and can only be estimated from measured data. Estimation of the health parameters from test data is often referred to as *gas path analysis* (GPA), which was pioneered by Urban [1]. The method of least-squares [2] and Kalman filters [3,4] are widely used for GPA. More recently, several artificial intelligence techniques such as neural networks [5], fuzzy logic [6], and Bayesian networks [7] were researched to be applied to GPA.

No matter which technique is used, there is a common difficulty in applying these techniques to an assessment of the condition of a gas turbine. When a health parameter estimator is built using one of these techniques, the estimator should be general enough to be applicable to various fault situations. However, a general estimator is not tailored to each fault situation so that its result may not be as accurate as the tailored ones. A too general estimator gives rise to the so called *smearing effect* [8] in its results. In a fault situation, the health parameter estimator should pinpoint the health parameters associated with the fault. The smearing effects refer to the spread of inaccuracy over several irrelevant health parameters.

The problem of choosing the generality of the estimator is an example of the concept called *model comparison* or *model selection* [9], which is well known in statistics. It is often found that, instead of selecting the single best model, combining multiple models gives rise to better results. In the field of fault diagnosis of gas turbines, there have been quite a few attempts to select the best model among many: the fault logic [2], a combinatorial approach [10], and a bank of Kalman filters [11]. However, there has been little effort to use the concept of *model combination* in this field.

Motivated by the potential of model combination, and inspired by several Bayesian approaches for selecting variables in regression analysis [12,13], the authors proposed the use of multiple Bayesian models using Bayesian model averaging (BMA) for fault diagnosis of gas turbines in a steady-state. The proposed method is intended for offline diagnoses. The inputs to the proposed method are sensor measurements obtained from a gas turbine. These measurements always contain random errors. In addition, biases can be present in measurements due to incorrect calibration or sensor faults. The random noises and biases in the measured data give rise to inaccuracy in estimates of health parameters. To avoid this inaccuracy, the measured data should be validated before they are used.

One of the simplest ways to remove random errors is to average multiple data points collected over a certain time period. Unlike random errors, biases are constantly present in multiple data points so that it is impossible to eliminate them by a simple manipulation of data. Because of its constant presence, biases can be misinterpreted as an actual change in the condition of the gas turbine or a fault situation. Unless the condition of the gas turbine is known, it is difficult to decouple biases from measured data [14]. Therefore, while the condition of the gas turbine is assessed, sensor biases must be considered as well. The Bayesian models used in this work explicitly include sensor biases, health parameters, and sensor measurements. Section 2 explains how multiple Bayesian models are constructed for various fault situations, and how their results are averaged. It is followed by the results from an industrial gas turbine fault diagnosis case.

2 Methodology

2.1 A Bayesian Model. Let X be a vector of health parameters, and Y as a vector of measurements. At a steady operating condition, the health parameters and measurements have a functional relationship f

$$Y = f(X) + \varepsilon \tag{1}$$

where ε is the random error. The relationship f can be linearized at the steady condition and written as

$$Y = AX + \varepsilon \tag{2}$$

where A is the coefficient matrix. Now ε includes not only the random noise but the linearization error as well. When the measurement vector Y is subject to sensor bias B, Eq. (2) can be written as follows:

$$Y = AX + B + \varepsilon \tag{3}$$

Let us assume that the conditional probability of Y, given X and B, $p(Y|X,B)$, follows a multivariate normal distribution $N(\mu,\tau)$, where μ is the mean vector, and τ is the precision matrix. Let us also assume that the mean of ε is zero. The mean vector μ is written as

$$\mu = AX + B \tag{4}$$

With limited data and knowledge, the precision matrix τ is often hard to define. Thus, τ is considered as a variable that is to be inferred from the data. A Wishart distribution is used for the prior distribution of τ as
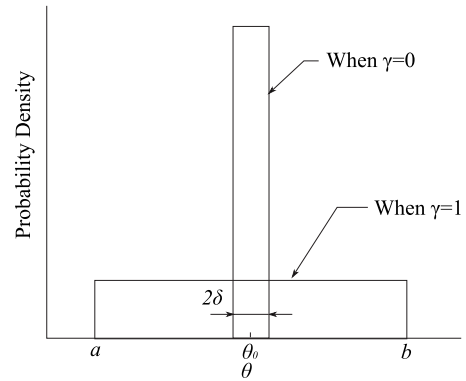

Fig. 1 "Spike and slab" prior

$$\tau \sim W(\Lambda, \nu) \tag{5}$$

where Λ is the scale matrix, and ν is the degree of freedom. A Wishart distribution is a conjugate prior of the precision matrix of a multivariate normal distribution [15]. The elements of Λ are adjusted to make the Wishart distribution disperse.

Health parameter X_i is assumed to be any value in the range of interest. It is also assumed that no particular value is more likely than others in the range. This notion can be expressed with a uniform distribution $U(a,b)$, where a and b are the lower and upper boundaries, respectively. With the same reason, the bias B is assumed to follow a uniform distribution as well. In reality, for a brand new machine, health parameters are likely to be the design condition, and even for an aged machine, a large deviation from the design condition may rarely happen. However, prior probability distributions, based on this conservative view, will affect the sensitivity and accuracy of a fault diagnosis algorithm.

2.2 Building Multiple Models. Let θ be a set of X and B, $\theta = \{X_1, X_2, \ldots, X_{nx}, B_1, B_2, \ldots, B_{nb}\}$, where nx and nb are the numbers of health parameters and sensor biases, respectively. The most general linear model one can build using any elements of θ is the one with all the health parameters and biases. Or, one can use a subset of the health parameters and biases such that the resulting model is tailored to a fault situation. The total number of possible subsets is 2^n, where n is the total number of health parameters and sensor biases. Each model is assigned to a state of model variable M. In reality, a situation with several faults may be less likely to happen than a situation with single or a few faults. However, it is objective to assign an equal probability to each model unless there is a sufficient reason to favor one model over another. Thus, a uniform categorical distribution with 2^n categories is assigned to the model variable M. The probability that the model variable M is a particular model m is

$$p(m) = \frac{1}{2^n} \tag{6}$$

As the model variable changes from one model to another, some variables are added in, and others are removed from the model. To implement this inclusion and exclusion of variables in a model numerically, an auxiliary variable vector γ is introduced, and its elements are connected to each element of θ. γ controls a mixture of two uniform distributions shown in Fig. 1, and assigns the mixture to θ as follows:

$$\theta | \gamma \sim (1 - \gamma) U(\theta_0 - \delta, \theta_0 + \delta) + \gamma U(a,b) \tag{7}$$

where θ_0 is the prescribed value, and δ is a small number. Each element of γ is a binary variable with two states: zero and one. When γ is zero, the uniform prior of the corresponding θ is concentrated at the prescribed value θ_0. On the other hand, when γ is one, the uniform prior covers the range of interest $[a,b]$. Thus,
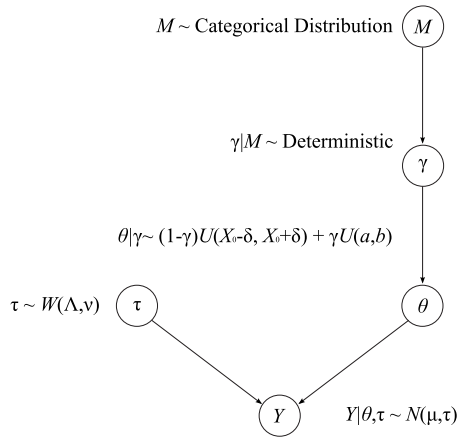
Fig. 2   Graphical model of the current formulation

any values in this range are equally probable. This emulates the inclusion and exclusion of θ connected to γ. Uniform distributions can be replaced with other distributions such as normal distributions. The current method is still valid with other distributions.

All 2^n models can be shown in one graphical form, as in Fig. 2. It should be noted that the relationship between M and γ is deterministic; the form of each model is already known.

2.3 Bayesian Model Averaging. Once the measurement vector Y is obtained from a gas turbine, it can be used for calculating the posterior distributions of other variables such as M and θ. The posterior of θ, $p(\theta|Y)$, can be calculated from

$$p(\theta|Y) = \sum_{m \in M} p(\theta|m,Y)p(m|Y) \propto \sum_{m \in M} p(Y|\theta,m)p(\theta|m)p(m)$$

(8)

Equation (8) is merely a weighted average of $p(\theta|m,Y)$, which resulted from each model. The posterior of each model $p(m|Y)$ is the weighting factor. All models are averaged through Eq. (8).

The model posterior $p(m|Y)$ can be calculated from

$$p(m|Y) \propto p(Y|m)p(m) \propto \int_{\Theta} p(Y|\theta,m)p(\theta|m)d\theta$$

(9)

Equation (9) involves a multidimensional integration of a possibly multimodal function. Because the integrand is not analytically derivable, the posterior should be calculated approximately, for example, using a Markov chain Monte Carlo (MCMC) method. MCMC methods are a class of algorithms for sampling from probability distributions. Among the MCMC methods, the Gibbs sampler is most widely used for Bayesian data analysis. The general formulation of the Gibbs sampler is as follows. Consider a vector of n random variables θ, another random variable Y, and the conditional density of i-th element of θ given Y, $p(\theta_i|\theta_1,\ldots,\theta_{i-1},\theta_{i+1},\ldots,\theta_n,Y)$. The Gibbs sampler sequentially samples from the conditional densities [16]

$$\theta_1^{j+1} \sim p(\theta_1|\theta_2^j,\theta_3^j,\ldots,\theta_n^j,Y)$$
$$\theta_2^{j+1} \sim p(\theta_2|\theta_1^{j+1},\theta_3^j,\ldots,\theta_n^j,Y)$$
$$\vdots$$
$$\theta_n^{j+1} \sim p(\theta_n|\theta_1^{j+1},\theta_2^{j+1},\ldots,\theta_{n-1}^{j+1},Y)$$

(10)

The symbol "$\sim$" means that θ_i^j follows the probability density p. The Gibbs sampler starts with an arbitrary initial θ^1, and randomly chooses θ_1^2 from the conditional density in the first equation. The chosen value for θ_1^2 is used in the rest of the equations at $j=1$.

Table 1 Health parameters and measurements

Health parameters	Measurements
Compressor efficiency (X_{CE})	Generator output (Y_{DW})
Compressor flow (X_{CF})	Compressor discharge temperature (Y_{CDT})
Turbine flow (X_{TF})	Compressor discharge pressure (Y_{CDP})
Turbine efficiency (X_{TE})	Exhaust gas temperature (Y_{TEX})
	Fuel flow (Y_{WF})
	Air flow (Y_{WA})

Throughout the sampling, Y is fixed. For large enough j, θ_i^j is effectively a sample point from its posterior distribution $p(\theta_i|Y)$. The number of required sampling varies with problems. For this paper, 3000 sampling are performed with an arbitrary initial point, and the sampling is repeated with another initial point. The conditional densities for the presented method are given in Fig. 2 along with the graph. WinBUGS [17], a public domain Gibbs sampling software, is used in this paper.

3   Test Case: An Industrial Gas Turbine

As a proof of concept, the presented method is applied to the fault diagnosis of a GE 7FA+e single shaft gas turbine. Four health parameters are to be estimated from six measurements, which are listed in Table 1. In addition to the health parameters, six biases, one for each measurement, are to be estimated as well. Typically, the bias in a sensor measurement is independent to other sensor measurements. However, the biases in the compressor discharge pressure and exhaust gas temperature measurements (B_{CDP} and B_{TEX}) affect all measurements because they are used for controlling the gas turbine. A Bayesian model consisting of all the health parameters, sensor biases, and measurements is shown in Fig. 3. This graph is corresponding to the link between θ and Y in Fig. 2.

The strength of each link in Fig. 3 is stored in the coefficient matrix A and in the coefficients of the sensor bias terms in Eq. (3). They can be determined by performing a regression analysis on available data. For this test case, a design of experiments (DOE) using thermodynamic analyses are performed. The Gas Turbine Performance (GTP) software [18], developed at GE, is used for simulating the gas turbine. The ranges of the health parameters and biases used in the DOE are shown in Table 2. A health parameter of 1 means that the performance of the corresponding component is same as the performance at the design condition. All sensor biases are shown in percentage of the values at the design condition except for the temperature sensor biases, which are in deviation from the design condition.

The developed method is applied to a fictional situation, as described in the following. The GE 7FA+e gas turbine experiences a fault or multiple faults so that the performance of the compressor is deteriorated. Both the compressor efficiency and flow parameters are 0.96. In addition to the compressor deterioration, the fuel flow sensor has 5% bias from the design condition
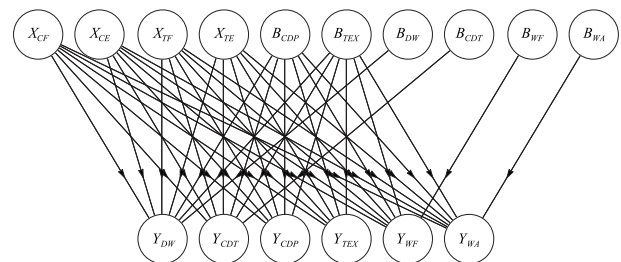

Fig. 3   Network consisting of the health parameters, sensor biases, and measurements

Table 2 Ranges of the health parameters and biases

Variables	Ranges
X_{CF}	$[0.92, 1.02]$
X_{CE}	$[0.92, 1.02]$
X_{TF}	$[0.96, 1.02]$
X_{TE}	$[0.95, 1.02]$
B_{CDP}	$[-2.5\%, 2.5\%]$
B_{TEX}	$[-44\,^{\circ}F, 44\,^{\circ}F]$
B_{DW}	$[-3\%, 3\%]$
B_{CDT}	$[-24\,^{\circ}F, 24\,^{\circ}F]$
B_{WF}	$[-10\%, 10\%]$
B_{WA}	$[-8\%, 8\%]$

value. All other components remain at the design condition. The measurements in this situation are simulated using GTP. To emulate random noises in real measurements, Gaussian random numbers with zero mean are added to the GTP output. The variances of the random noises of the measurements are obtained from the test data provided by GE Energy, and they are not listed here due to their proprietary nature.
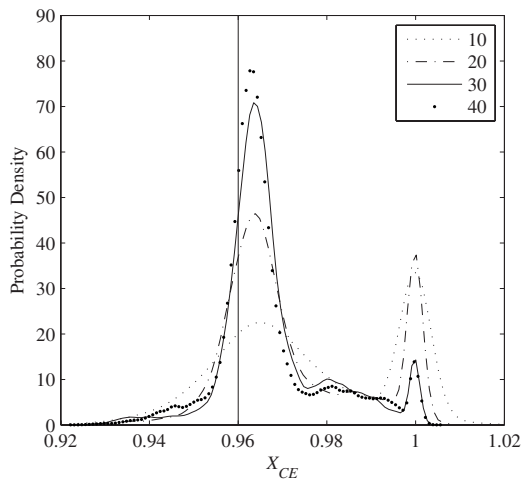

Fig. 4 Posterior of the compressor efficiency parameter with various numbers of data points (vertical lines: true values)
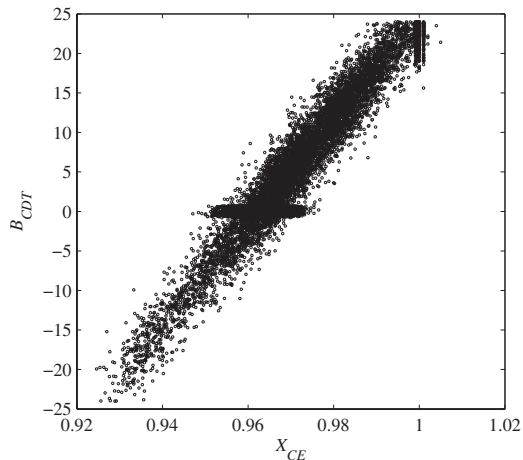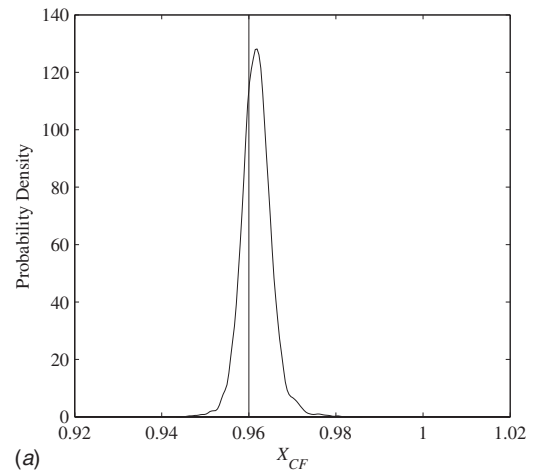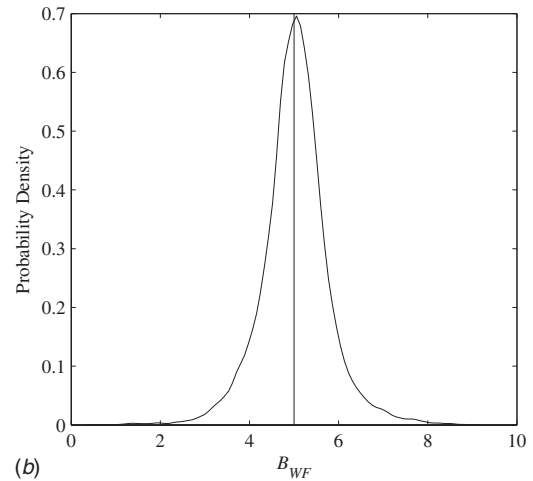

Fig. 5 Gibbs samples in the X_{CE}-B_{CDT} coordinate

Fig. 6 Posteriors of the compressor flow parameter and the fuel flow sensor bias (vertical lines: true values)

The data fed into the method is a two-dimensional matrix, each row of which is a vector of six measurements obtained at different discrete instants of time. In reality, each data point is often a time average of a short period, and the interval between these data points can be as short as a few minutes. However, the time interval is irrelevant in this paper because of the steady-state assumption.
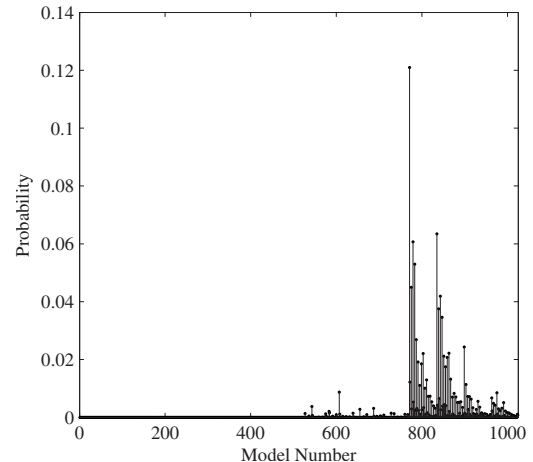

Fig. 7 Model posterior distribution

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Table 3 Models with the ten highest posterior probabilities

Probability	Cumulative probability	Variables									
		X_{CF}	X_{CE}	X_{TF}	X_{TE}	B_{CDP}	B_{TEX}	B_{DW}	B_{CDT}	B_{WF}	B_{WA}
8.52	8.52	√	√							√	
4.94	13.46	√	√					√	√	√	
4.54	18.01	√	√					√		√	
3.43	21.44	√	√		√			√		√	
3.40	24.84	√	√		√					√	
3.08	27.92	√	√		√			√	√	√	
2.93	30.86	√	√						√	√	
2.87	33.72	√	√		√				√	√	
2.36	36.08	√	√				√			√	
2.05	38.13	√	√			√				√	

A solution from the method varies with the number of data points. When the number of data points is small, the prior distributions of the variables will dominate the solution. As the number of data points increases, the effect of the prior distributions will be diminished. To find a proper number of data points, the method is applied to the test case with various numbers of data points, and Fig. 4 shows the posterior of the compressor efficiency parameter. The posterior of the compressor efficiency parameter is multimodal: one mode near 0.96, which is the true value, and the other near 1. The latter is due to the models that do not include the compressor efficiency parameter. When a health parameter is not included in a model, the health parameter is fixed at 1 as default, which is the design condition. As the number of data points increases, the density near 1 decreases, and the decreased amount shifts near the true value. After 30 data points, the posterior changes a little. The results shown hereafter are from the case of the 30 data points. The computational time is typically within five minutes on a PC with Intel Core2 2 GHz CPU and 2 GB of RAM.

This multimodality is due to the correlation between X_{CE} and B_{CDT}, given the measurements for this test case. The correlation can be seen once the 6000 Gibbs samples are plotted in the X_{CE}-B_{CDT} coordinate, as shown in Fig. 5. The horizontal and vertical clouds at $X_{CE}=0.96$ and 1 are from the models in which B_{CDT} and X_{CE}, respectively, are not included. These two clouds on top of the other points make the probability density of X_{CE} peaked at 0.96 and 1. It should be noted that correlations between the variables vary with the coefficient matrix and measurements.

Figure 6 shows the posteriors of the compressor flow parameter and the fuel flow sensor bias. Unlike the compressor efficiency parameter, both the compressor flow parameter and the fuel flow bias are unimodal because these two do not have any significant correlation with other variables. Each posterior is peaked near the true value. Thus, an accurate point estimate such as the maximum a posterior (MAP) estimate can be made from these posteriors. All the variables not presented here also have posteriors peaked at the true values.

The results so far are an average of results from multiple models. Because the total number of health parameters and sensor biases is ten, $2^{10}=1024$ models are considered for this analysis. More than half of these 1024 models have a nearly zero posterior probability, as shown in Fig. 7. These models with a nearly zero posterior probability are barely supported by the data, and they contribute little to the posterior of θ in Eq. (8).

Table 3 lists the ten models most supported by the data. All ten models commonly contain the compressor efficiency, flow parameters, and the fuel flow sensor bias. Although the developed method finds the model with only the necessary variables, the most probable in this test case, in general, it is possible for other similar models to have higher posterior probability due to the limited amount of data and the noise in data. In fact, the top ranked model is the model tailored to the fault situation in this test case. It should perform the best among the 1024 models in this test case. The most general model among the 1024 models is the one with all the health parameters and biases as its variables. Hereafter, the tailored model and the most general models will be referred to as the *true* and *full* models, respectively. Intuitively, the averaged model is expected to perform in between the two models. Figure 8 shows the posteriors of the compressor efficiency and flow parameters from the true, full, and Bayesian averaged models. The true model results in the most peaked and closest posteriors to the true value. In contrast, the full model results in the disperse posteriors for both parameters, which are not much different from their uniform prior distributions. It is hard to draw
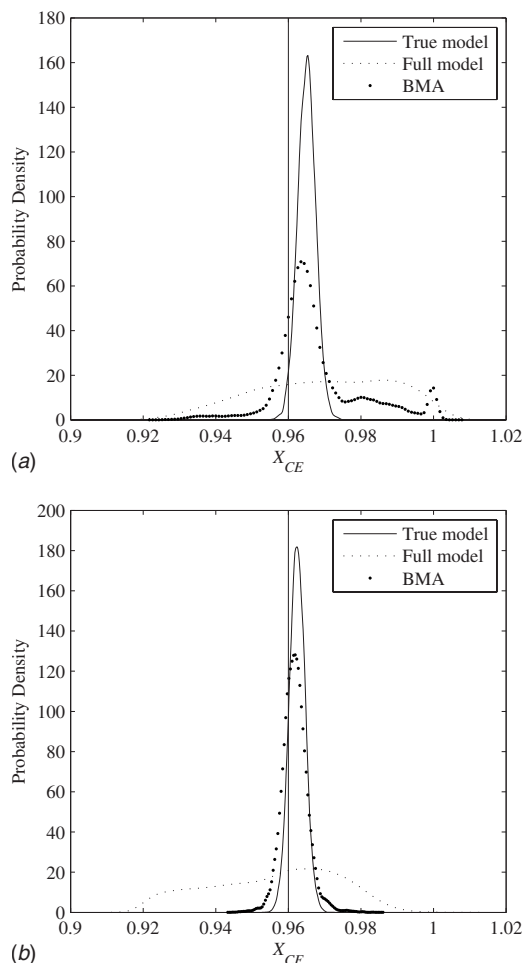

Fig. 8 Comparison of the true, full, and Bayesian averaged models (vertical lines: true values)

a conclusion or make an accurate point estimate from these disperse distributions. Last, but not the least, the averaged model approximates the true model fairly well, and the posteriors are peaked enough to make a meaningful point estimate.

4 Conclusion

This paper presents an offline fault diagnosis method for industrial gas turbines in a steady-state. Multiple Bayesian models tailored to various fault situations are implemented in one hierarchical model. The fault situations include single faults occurring in a component, and multiple faults occurring in more than one component. In addition to faults occurring in the components of a gas turbine, sensor biases are explicitly included in the Bayesian models. Results from these multiple Bayesian models are averaged using the posterior probability of each model as a weighting factor. The Gibbs sampling is used to calculate approximate posterior probability distributions.

The presented method is applied to fault diagnosis of a GE 7FA+e single shaft gas turbine that is equipped with the faulty compressor and the biased fuel flow sensor. The presented method successfully detects and identifies the magnitudes of the compressor fault and the fuel flow sensor bias with a limited amount of data. It is also shown that Bayesian model averaging gives rise to more accurate and less uncertain results than a single complex model.

Acknowledgment

Nomenclature

Y = vector of sensor measurements
X = vector of health parameters
B = vector of sensor biases
ε = random noise with zero mean
$W(\Lambda, \nu)$ = a Wishart distribution with the parameters Λ and ν
$U(a, b)$ = a uniform distribution with the range between a and b
$N(\mu, \tau)$ = a normal distribution with the mean vector μ and the precision matrix τ
M = model variable
m = an instance of the model variable
θ = a set of the health parameters and sensor biases
γ = vector of binary variables controlling the inclusion or exclusion of variables

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